

From Megan Lionberger

Mass Balance Error

Tetra Tech, Inc. tested the Uncles-Peterson sediment-PCB model by scaling multiple model variables by +/-10 percent. The scaled model variables of the sediment transport component included 3 calibration coefficients for wind shear, erosion rate, and deposition rate, and sediment loads from both the Delta and local Tributaries. In 5 of 100 runs, the model produced an error that said:

i = 2 j = 1 dep = 1.00 (or higher)
error in sediment mass conservation

This error indicates that the model is attempting to deposit more sediment than is in suspension in the lower layer (i = 2) of model segment 1 (j = 1). The value dep is the ratio of the calculated mass of sediment to be deposited to the mass of sediment in suspension. When the dep ratio is greater than 1.00, the model produces this error to alert the model user and then reassigns the mass to be deposited to equal the mass of sediment in suspension. Therefore, mass is still being conserved even when the error message is displayed. The model user hits the return key and the model will continue to run. Typically, the error message is produced multiple times for a single day because the time step is subdivided.

Each daily time step is subdivided for each segment so that no more than 25 percent of sediment in suspension is deposited at a time. The time step subdivision is calculated once at the beginning of the day based on the initial suspended-sediment concentration of the segment. The daily time step can be subdivided into any number of sub-time steps. This was necessary to be able to increase both the erosion and deposition calibration coefficients while still conserving mass.

Model segments are composed of two layers, an upper layer representing the shoals and a lower layer representing the channel. Segment 1 represents the shallowest part of the estuary with an initial lower-layer depth of 0.5 meter. After its initial spin-up time, the equilibrium depth reduces to less than 0.1 meter. The error message was produced on the same day for each failed simulations for the lower layer of segment 1 corresponding to the second largest daily-tributary load over the 60 year model run. The error was caused by high concentrations of suspended-sediment depositing to the shallow lower layer, reducing its volume until more sediment is being deposited than the layer has capacity for.

The way the model was coded, the bed shear was calculated only once at the beginning of the time step for each segment. Bed shear in the lower layer is calculated as

$$\tau_b = \frac{g\rho u^2}{C_z^2} \quad (1)$$

where

g = gravity, m/s²

ρ = density, kg/m³

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\bar{u} = tidally averaged current speed, m/s

C_z = Chezy coefficient, $m^{1/2}/s$

c_w = wind calibration constant

The Chezy coefficient is calculated as

$$C_z = \frac{h^{1/6}}{n} \quad (2)$$

where h is the average water depth and n is Manning's roughness parameter. As the water depth decreases, bed shear increases, also increasing the erosion rate. In this way, sediment is not allowed to deposit indefinitely and is constrained within the volume of the segment layer. Because the bed shear was only calculated once at the beginning of the time step, the model wasn't eroding enough of the newly deposited sediment in the lower layer of segment 1. I moved the bed shear calculation from outside to inside the subdivided time stepping loop to recalculate the bed shear for every subdivided time step. The 5 failed model simulations were rerun without producing a mass conservation error. Continuously recalculating the bed shear throughout the subdivided time step, based on the change in bed height, was able to stabilize the model.

To convince myself that model would continue to run without mass balance errors, I performed a not-so-random variable scaling analysis. Ten simulations were run varying coefficients by +/-20% (table 1). None of the simulation runs produced a mass balance error.

Run	CONSTEF	CONSTWF	CONSTDF	SEDTRIB	SEDDELT
1	0.8	0.8	1.2	1.2	1.2
2	0.9	0.9	1.1	1.1	1.1
3	1.2	1.2	0.8	0.8	0.8
4	1.1	1.1	0.9	0.9	0.9
5	1.2	1.2	1.2	1.2	1.2
6	0.8	0.8	0.8	0.8	0.8
7	0.8	0.9	1.0	1.2	1.2
8	1.2	1.0	0.8	1.0	1.0
9	0.9	1.1	0.9	1.1	0.9
10	1.1	0.8	1.2	1.2	1.1

Table 1. Scaling coefficients, as defined by John Oram, used in the model simulations.

The model did not need to be recalibrated. Moving the location of the bed shear calculation had only a minor effect on model outputs.